Engines course

Exercise I: Thermodynamic cycle

Solution:

1) Isentropic relations : $\frac{P_2}{P_1} = \left(\frac{v_1}{v_2}\right)^{\gamma} = \varepsilon^{\gamma}$ and $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\varepsilon^{\gamma}\right)^{\frac{\gamma-1}{\gamma}} = \varepsilon^{\gamma-1}$

State 2:
$$P_2 = P_1 \cdot \varepsilon^{\gamma}$$
$$T_2 = T_1 \cdot \varepsilon^{\gamma-1}$$
$$v_2 = v_1 \cdot \varepsilon^{-1}$$

with: $q_{comb} = c_v \cdot (T_3 - T_2) \quad \rightarrow \quad \frac{T_3}{T_1} = \underbrace{\frac{q_{comb}}{c_v \cdot T_1}}_{\xi} + \frac{T_2}{T_1} = \xi + \varepsilon^{\gamma-1}$

and: $\frac{P_3}{P_1} = \frac{T_3 \cdot v_1}{T_1 \cdot v_3} = \frac{T_3}{T_1} \cdot \varepsilon$

State 3:
$$P_3 = P_1 \cdot \varepsilon^{\gamma} \cdot \left(1 + \xi \cdot \varepsilon^{1-\gamma}\right)$$
$$T_3 = T_1 \cdot \varepsilon^{\gamma-1} \cdot \left(1 + \xi \cdot \varepsilon^{1-\gamma}\right)$$
$$v_3 = v_1 \cdot \varepsilon^{-1}$$

with: $\frac{P_4}{P_3} = \left(\frac{v_3}{v_4}\right)^{\gamma} = \varepsilon^{-\gamma}$ and $\frac{T_4}{T_3} = \left(\frac{P_4}{P_3}\right)^{\frac{\gamma-1}{\gamma}} = \varepsilon^{1-\gamma}$

State 4:
$$P_4 = P_1 \cdot \left(1 + \xi \cdot \varepsilon^{1-\gamma}\right)$$
$$T_4 = T_1 \cdot \left(1 + \xi \cdot \varepsilon^{1-\gamma}\right)$$
$$v_4 = v_1$$

2) Summary table along the thermodynamic process (state 1 to 4) :

	Transform.	Work	Heat
	$1 \rightarrow 2$	${}_1^2e = u_2 - u_1 = c_v \cdot (T_2 - T_1)$	${}_1^2q = 0$
	$2 \rightarrow 3$	${}_2^3e = 0$	${}_2^3q = q_{comb} = u_3 - u_2 = c_v \cdot (T_3 - T_2)$
	$3 \rightarrow 4$	${}_3^4e = u_4 - u_3 = c_v \cdot (T_4 - T_3)$	${}_3^4q = 0$
+	$4 \rightarrow 1$	${}_4^1e = 0$	${}_4^1q = u_1 - u_4 = c_v \cdot (T_1 - T_4)$
	Total	$e_{cycle} = c_v \cdot (T_2 - T_1 + T_4 - T_3)$	$q_{cycle} = c_v \cdot (T_3 - T_2 + T_1 - T_4)$

1st Law: $\Delta u_{cycle} = e_{cycle} + q_{cycle} = 0$ or $\oint_{cycle} du = 0$

Computation of the mechanical work: $e_{cycle} = -q_{cycle} = \oint_{1=>1} \delta e = e_1^2 + e_2^3 + e_3^4 + e_4^1$

according to the table above, this expression gives:

$$-e_{cycle} = q_{cycle} = c_v \cdot (T_3 - T_2 + T_1 - T_4)$$

$$= c_v \cdot T_1 \cdot \left(\frac{T_3}{T_1} - \frac{T_2}{T_1} + 1 - \frac{T_4}{T_1}\right)$$ with the relations obtained in part 1,

$$= c_v \cdot T_1 \cdot \left(\varepsilon^{\gamma-1} \cdot \left(1 + \xi \cdot \varepsilon^{1-\gamma}\right) - \varepsilon^{\gamma-1} + 1 - \left(1 + \xi \cdot \varepsilon^{1-\gamma}\right)\right)$$

$$= c_v \cdot T_1 \cdot \left(\varepsilon^{\gamma-1} + \xi - \varepsilon^{\gamma-1} + 1 - 1 - \xi \cdot \varepsilon^{1-\gamma}\right)$$

$$-e_{cycle} = \underbrace{\xi \cdot c_v \cdot T_1}_{q_{comb}} \cdot \left(1 - \varepsilon^{1-\gamma}\right)$$

Applying the relation of thermodynamic efficiency given by: $\eta = \dfrac{-e_{cycle}}{q_{comb}}$

$$\Rightarrow \quad \eta = 1 - \frac{1}{\varepsilon^{\gamma-1}} \quad \text{with} \quad \varepsilon = \frac{V_{BDC}}{V_{TDC}}$$

The efficiency is directly and only depending on the engine's compression ratio !